Class: 2nd Year → Definitions From Text Book

Chapter: 1

Functions

1. **Function:**

If A and B be two non-empty sets then f is said to be a function from set A to set B written as ;$f: A \to B$ and defined as

$$i) D_f = A \; ii) \; for \; every \; a \in A \; thereexist \; only \; one$$

$b \in B \; s.that \; (a, b) \in f$

2. **Domain:**

The set of all possible inputs of a function is called domain.

*the domain of every function $f(x)$ is defined.

*the valves at which at $f(x)$ becomes undefined or complex valued will be excluded from real numbers.

*domain is also known as pre-images.

3. Range:

The set of all possible out puts of a function is called range.

*range is also known as images.

3. **Algebraic function:**

Any function generated by algebraic operations is known as algebraic function. Algebraic functions are classified as below.

4. **Polynomial function:**

A function P of the form

$$p(x) = a_n x_n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2 + a_1x + a_0$$
$$for \; all \; x, where \; the \; coefficients \; a_n, a_{n-1}, a_{n-2}. a_2, a_1, a_0$$
$$are \; real \; numbers \; and \; exponents \; are \; non-negative$$

$integers, is \; called \; a \; polynomial \; function.$

5. **Linear Function:**

If the degree of polynomial function is 1. Then it is called linear function.

6. **Quadratic Function:**

If the degree of polynomial function is 2. Then it is called a quadratic function.

7. Identity function:

A function for which $f(x) = y \; or \; y = x$ is called identity function. It is denoted by I

8. **Constants Function:**

A function for which $f(x) = b \; or \; y = b$ is called constant function.

9. **Rational function:**

The quotient of two polynomials such as $f(x) = \frac{p(x)}{Q(x)}$

$where \; Q(x) \neq 0 \; is \; called \; rational \; function$

10. **Exponential Function:**

A function in which the variable appears as exponent (power) is called exponential function.

$e.g; y = e^{ax}, y = e^x \; e.t.c$

11. Logarithmic Functions:

$if\ x = a^y\ then\ y = loga^x\ where\ a > o, a \neq 1\ is\ called\ logarthmic\ functions.$

*$\log_{10} x$ $is\ knonw\ as\ common\ logarithm.$

*$\log_e x\ is\ known\ as\ natural\ logarithm.$

12. Explicit function:

If y is easily expressed in terms of x, then y is called explicit function.

$symbolically\ y = f(x)$

13. Implicit function:

If the two variables x and y are so mixed up such that y cannot be expressed in terms of x, then this type of function. Symbolically$f(x, y) = 0$

14. Parametric function:

If x and y are expressed in terms of third variable (say t) such as $x = f(t), y = g(t)\ then\ these\ equations\ are$ Called parametric equations.

15. Even function:

A function f is said to be an even if $f(-x) = f(x)$ for every $x\ in\ domain\ of\ f.$

16. **Odd function**:

A function f is said to be odd if $f(-x) = -f(x)$ for every number x in the domain of f

17. Composition of function:

If f is a function from set A to set B and g is a function from set B to set C then composition of f and g is denoted by

$$(fog)(x) = f\big(g(x)\big) \forall\ x \in A$$

18. Inverse of a function:

Let f be a bijective $(1-1\ and\ onto\ )function\ from\ set\ \ A\ set\ b\ i.e\ f: A \to B\ then\ its\ inverse\ is\ f^{-1}\ which\ is\ \ surjective\ (onto)function\ from\ B\ to\ A\ i.e\ \ f^{-1}: B \to A\ in\ this\ case\ D_f: R_f\ on\ to\ R_f = D_{f^{-1}}$

19. The left hand limit:

$$if\ \lim_{x \to a^-} f(x) = L\ it\ means\ f(x) takes\ value\ L\ as\ x$$

$approaches\ to$ a$from\ the\ left\ side\ of\ "a"$
$(i.e\ from\ -\infty\ to\ a) then\ \lim_{x \to a^-} f(x) = L\ is\ called$
$left\ hand\ limit.$

20. *The Right hand limit*:

$$if\ \lim_{x \to a^+} f(x) = L\ it\ means\ f(x) takes\ value\ L\ as\ x$$

$$approaches\ to\ a from\ the\ right\ side\ of\ a(i.e\ from$$

$a\ to\ \infty)\ then\ \lim_{x \to a^+} f(x) = L\ is\ called\ right$
$hand\ limit.$

Chapter 2

Integration

1. **Average rate of change**

Let f $be\ a\ real\ valued\ function\ the\ (difference\ quotient)$

$\frac{f(x_1)-f(x)}{x_1-x}$ $is\ called\ average\ rate\ of\ cahnge.$

2. **Derivative:**

$let\ f(x) be\ a\ function, then\ its\ derivative\ is\ denoted$ $by\ f'(x) or \frac{df}{dx}$ $and\ defined\ as;$

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

- The process of finding f' is called differentiation.

3. **Derivative of exponential Functions:**

   $if\ f(x) = a^x\ where\ a > 0\ then\ f(x) is\ called\ general$ expontial function. Here the base a is constant and exponent x is variable.

4. **Taylor Series:**

$The\ series$

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \cdots + \frac{f^n(x)}{n!}h^n$$

Is known as **"Taylor series expansion"**

5. **Increasing functions:**

A function f is said to be increasing function on an interval (a, b) if for every $x_1, x_2 \in (a, b)\ then$ $f(x_1) < f(x_2)\ whenever\ x_1 < x_2$

6. **Decreasing function:**

A function f is said to be decreasing function on an interval (a, b) if for every $x_1, x_2 \in (a, b)\ then$ $f(x_1) > f(x_2)\ whenever\ x_1 < x_2$

7. **Constant function:**

A function f is said to be constant function on an interval (a, b) if for every $x_1, x_2 \in (a, b)\ then$ $f(x_1) = f(x_2)\ whenever\ x_1 < x_2$

8. **Increasing functions**:

A differentiable function f is said to be increasing function $if\ f'(x) > 0\ \forall x \in (a, b)$

9. **Decreasing function:**

A differentiable function f is said to be decreasing function $if\ f'(x) < 0\ \forall x \in (a, b)$

10. **Constant function:**

A differentiable function f is said to be constant function $if\ f'(x) = 0\ \forall x \in (a, b)$

11. **Stationary point:**

Any point where f is neither increasing nor decreasing is called stationary point.
Provided that $f'(x) = 0\ at\ that\ point.$

12. Relative Maxima:

A function $f(x)$ has relative maxima $f(c)$ at

$x = c \in (a,b) if$

$$(i) f(c-\delta x) > 0 \quad (ii) f'(c) = 0 \quad (iii) f'(c+\delta x) < 0$$

$where\ \delta x\ is\ small + ve\ is\ small\ number.$

13. Relative Minima:

A function $f(x)$ has relative minima $f(c)\ at$

$x = \ c \in (a,b) if$

$$(i) f(c-\delta x) < 0 \quad (ii) f'(c) = 0 \quad (iii) f'(c+\delta x) > 0$$

$where\ \delta x\ is\ small + ve\ is\ small + ve\ number.$

14. Relative maxima:

A function $f(x)$ has relative maxima $f(c) if\ f'(x)\ chnages\ sign\ from + ve\ to - ve\ as\ x\ change\ through\ C.$

15. Relative minima:

A function $f(x)$ has relative minima $f(c) if\ f'(x)\ chnages\ sign\ from - ve\ to - ve\ as\ x\ change\ through$ C

16. Relative Extrema:

Both relative maxima and relative minima are called in general "relative extreme"

17. Turning point:

A stationary point is called a turning point or a minimum point.

18. Point of inflexion:

A point at which the function has neither maximum nor minimum value is called point of inflexion.

Chapter 3

INTEGRATION

1. The technique or method to find such a function whose derivative is given involves the inverse process of differentiation, called anti derivative or integration.
2. $The\ inverse\ process\ of\ differentiation\ is\ called\ anti - differentiation\ or\ integration.$
3. **The Definite integrals:**

   If $\int f(x)dx = \emptyset(x) + c, then\ the\ integral\ of\ f(x)\ from\ a\ to\ b\ is\ denoted\ by\ \int_a^b f(x)dx$

   And read a+cs definite integral of $f(x)$ here a is called lower limit and b is called upper limit.

   *the interval $[a,b]$ is called range of integration.
4. **Fundamental theorem of calculus:**

   If $f(x)$is continuous $\forall\ x \in [a,b]\ and\ \emptyset'(x) = f(x)\ \int_a^b f(x)dx \ = \emptyset(b) - \emptyset(a)$
5. $\boldsymbol{Differential\ equation}$:

   An equation containing at least one derivative of a dependent variable with respect to an independent variable is called differential equation. $e.g.$

   $$y\frac{dy}{dx} + 2x = 0 \qquad and\quad x\ \frac{d^2y}{dx^2} + \frac{dy}{dx} - 2x = 0$$
6. $\boldsymbol{Order\ of\ Differential\ equation}$:

$The\ order\ of\ the\ differential\ equation\ is\ the\ order\ of\ the\ highest\ derivative\ in\ the\ equation.$

$y\frac{dy}{dx} + 2x = 0 \quad (1st\ order\ differential\ equation)$

$x\,\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2x = 0(2nd\ \ order\ differential\ equation)$

7. $\boldsymbol{Degree\ of\ Differential\ equation}$:
   The degree of a differential equation is the greatest power of the highest order derivative in the equation.

   $x\,\frac{d^4y}{dx^4} + \frac{dy}{dx} - x\,\left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} + 2x = 0 \qquad (1st\ degree\ differential\ equation)$

   $x\,\left(\frac{d^4y}{dx^4}\right)^3 + \frac{dy}{dx} - x\,\left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} + 2x = 0 \qquad (3rd\ degree\ differential\ equation)$
8. $\boldsymbol{General\ solution}$:
   $The\ solution\ of\ differential\ equation\ which\ contains\ arbitrary\ constants\ is\ called\ general\ solution.$
9. $\boldsymbol{Particular\ solution}$:
   $The\ solution\ obtained\ from\ general\ solution\ by\ applying\ the\ initial\ conditions\ is\ called\ particular\ \ solution.$
10. $\boldsymbol{Initial\ value\ conditions}$:
    The arbitrary constants involving in the solution of a differential equation can be determine by the Seven conditions. Such conditions are called Initial value conditions.

Chapter: 4

Introduction to Analytic Geometry

1. **Geometry:**

   The geometry is derived from two Greek words Geo (Earth) and Matron (Measurement). It means Knowledge of measurement of earth.*Geometry is branch of mathematics that deals the shape and size of things.
2. **Analytic geometry:**

   In analytic geometry or coordinates geometry, points could be represented by numbers, lines and curves represented by equations.

   A French philosopher and mathematician Rene Descartes (1596-1650A.D) introduced algebraic methods in geometry named as analytical geometry named (or coordinate geometry.)
3. **Coordinates system:**

   Draw in a plane two mutually number lines $XX'and\ YY'$

   One horizontal and the other vertical. Let their point of intersection be O called origin and real number O of both lines is represented by O. The two lines are called the coordinate axis.
4. **Inclination of lines:** The angle $(O^0 < \alpha < 180^0)$ Measured anti-clockwise from positive

   $x - axis\ to$ A non-horizontal straight line $l\ is\ called$ inclination of l
5. **Slope or gradient of a line:**

   Let αbe inclination of a line, then slope of a line is denoted by m and denoted by m defined as

   $m = tan\alpha$

6. **Distance b/w two parallel lines**

The distance between two parallel lines is the distance from any point on one of the lines to the other.

7. **Homogenous Equation:**

Let $f(x,y) = 0 \rightarrow (i) be\ any\ equation\ in\ variables$
$x\ and\ y\ is\ called\ homogenous\ equation\ of\ degree\ n\ (a + ve\ integer) if$
$f(kx, ky) = k^n f(x,y) for\ k \in R$

Chapter: 5

Linear Inequalities and Linear Programming

1. **Linear Inequalities in one variable:**

The inequalities of the form $ax + b < 0, ax + b > c\ ax + b \geq c\ and\ ax + by \leq c\ are\ called\ \ linear\ inequalities$ In one variable.

2. **Linear inequalities in two variables:**

The inequalities of the form $ax + by + c,$
$ax + b > c\ and\ ax + by \geq c\ \ and\ ax + by \leq 0$
$are\ called\ linear\ inqualities\ in\ two\ variables$
$and\ y\ where\ a, b\ and\ c\ are\ constants.$

3. **Solution set of linear inequalities:**

The ordered pair $(a, b) which\ satisfy\ the\ linear$ inequality in two variables $x\ and\ y$ form the solution.

4. **Solution Region:**

Solution region of system of inequalities is the common region that satisfies all given inequality in the system.

5. **Corner point / vertex:**

A point of the solution region where two of its boundary lines intersect is called the corner point or vertex of the solution region.

6. **Problem constraints:**

The restrictions applied on the everyday life problems are called problem concentration.

7. **Non- Negative constraints:**

The constraints that are always satisfied are called natural constraints or non- negative constraints.

8. **Decision variable:**

The variable used in non- negative constraints are called decision variable.

9. **Feasible region:**

The solution region which is restricted to the first quadrant is called feasible region. We restricted the solution region by using non-negative constraints $x \geq 0\ and\ y \geq 0$

10. Feasible solution:

Each point of feasible region is called feasible solution of the system.

11. Feasible solution Set:

A set consists of all the feasible solution of the system is called feasible solution.

12. Objective function:

A function which is to be maximized or minimized is called an objective function:

13. Optimal solution:

The feasible solution which maximizes or minimize the objective function is called optimal solution.

Chapter: 6

Consequence

1. **Cone:**

A solid figure generated by a straight line passing through a fixed point and revolving about a fixed line is called cone.

2. **Axis of Cone:**

The fixed line passing through vertex and perpendicular to the center of base is called axis of cone.

3. **Apex:**

The fixed point of cone is called vertex or apex of cone.

4. **Napes of Cone:**

The cone has two parts called napes. The part above the vertex is upper nape. The part below the vertex is lower nape.

5. Cones are named according to shape of their base. If the base of cone is circle then it is called circular cone. If the base of cone is ellipse, then it called elliptic cone.

“ A cone having its axis perpendicular circular base is called Right circular cone.”

6. **Conic Section:**

Some standard conic sections are Circle, Ellipse parabola and Hyperbola.

We first study properties of a circle other conics will be taken up later.

7. **Circle:**

A set of all points in a plane which are equidistance from a fixed point is called circle. The fixed point is celled center and fixed distance is called radius of the circle.

8. **Points Circle:**

If the plane passes through vertex of cone, the instruction is a single point or if $r = 0$

9. Tangent to a circle:

"A line which touches the circle without cutting it, is called tangent line"

10. Normal to a circle:

"A Line $\perp$ *ar to the tangent line at a point of tandency*

is called Normal line. for circle $x^2 + y^2 = r^2$

$(\because x_1 + y_1 = r^2) by\ (ii)$

11.Equation of Normal

$$\because slope\ of\ normal\ line = \frac{y_1}{x_1}$$

so eq. of normal line at $P(x, y) is$

$$y - y_1 = \frac{y_1}{x_1}(x - x_1)$$

⇨ $x_1y - x_1y_1 = xy_1 - x_1y_1$

⇨ $x_1y = xy_1$

⇨ $\frac{y}{y_1} = \frac{x}{x_1}$

Which is eq. of normal line.

12. "Chord of a Circle:

A line segment where end points lie on a circle is called chord of a circle. In figure AB is chord.

13. Diameter of a circle:

A chord passing through Centre of a circle, if O is Centre the CD is diameter of circle.

14. Parabola:

Set of all points which are equidistance from a fixed point and a fixed line.

- Fixed point is called focus of parabola. Fixed line is called Directrix of parabola

15. The line through the focus and perpendicular to the *directrix* is called **axis of parabola.**

16. The point where the axis meets the parabola is called **vertex** of parabola.

17. In parabola, fixed point not on the *directrix* is called **focus of parabola.**

18. In parabola, fixed line is called ***directrix* of parabola.**

19.A line passing through vertex and perpendicular to axis of parabola **is called tangent at vertex of parabola.**

20.A line whose points lie on parabola is called chord of parabola.

21.A chord passing through focus of parabola is called **focal chord.**

22.A focal chord perpendicular to axis of parabola is called **Let rectum.**

23. The midpoint of line segment joining the foci is called Centre of hyperbola. In fig. $F\ and\ F'$ are foci and O is Centre.

24. The line passing through the foci of hyperbola is called focal axis or Transverse axis.

25. The line passing through Centre of hyperbola and perpendicular to transverse axis is called conjugate axis. In fig. y-axis is conjugate axis.

26. The points where the hyperbola meets its transverse axis are called vertices of hyperbola.in fig. $V\ and\ V'are\ vertices\ of\ hyperbola.$

27. Length of $leturectum\ is\ \frac{2b^2}{a}$

28. If the point $asec\theta, bTan\theta)$ lies on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\ where\ \theta \in R\ then\ x = asec\theta$

$y = atan\theta\ are\ parametric\ equations\ of\ hyperbola.$

Standard Equation of hyperbola

Chapter: 7

Vector

1. **Scalar:**

A quantity which has magnitude only is called scalar. $e.g$ time, volume, speed and work e.tc the scalars are denoted by letters.

2. **Vectors:**

A quantity which has both magnitude and direction is called vector. $e.g\ Velocity$, displacement, force and torque $e.t.c$. A vector $say\ V\ is\ denoted\ by\ \vec{V}\ or\ V$ e.t.c. or by bold face letter v.

3. **Geometrical interpretation of Vector:**

Geometrically, A vector is represented by a line segment AB with A its initial point and B its terminal point. It is often found convenient to denoted a vector by an arrow and written either as $\overrightarrow{AB}$ or as a bold face symbol like $v\ or\ in\ underlined\ form\underline{v}$.

4. **Magnitude of a vector.**

Let $v\ be\ a\ vector$, then its magnitude is denoted by $|\vec{v}|$

$or\ simly\ v.it\ is\ also\ called\ norm\ or\ length\ of\ vector$

$if\ the\ line\ \overrightarrow{AB}\ represent\ a\ vector, then\ distance\ from$

Point A to point B will be magnitude of $\overrightarrow{AB}\ and\ is$

Denoted by $|\overrightarrow{AB}|$

5. **Unit vector:**

A vector whose magnitude is one (unity) is called unit vector. Unit vector of $\underline{v}\ is\ written\ as\ \hat{v}$ (read as v hat) and is defined as $\hat{v} = \frac{\underline{v}}{|\underline{v}|}$

6. **Null vector:**

A vector whose magnitude is zero but no specific direction is called null or zero vector. It is denoted by $\vec{o}$

7. **Negative Vectors:**

Two vectors are said to be negative of each other if they have the same magnitude but opposite direction.

If $\overline{AB} = \underline{v}$ then $\overline{BA} = -\overline{AB} = -\underline{v}\ and\ |\overline{BA}| = |-\overline{AB}|$

$(\because\ the\ magnitude\ of\ a\ vector\ is\ a\ non\ negative\ member)$

8. **Equal Vectors:**

Two vectors $\overrightarrow{AB}\ and\ \overrightarrow{CD}\ are\ said\ to\ be\ equal, if\ they\ have\ same$

$magnitude\ and\ same\ direction\ i.e\ |\overrightarrow{AB}| = |\overrightarrow{CD}|$

9. Parallel vectors:

Two vectors $\vec{u}, and\ \vec{v}\ are\ said\ to\ be\ parallel\ if\ \vec{u} \times k\ \vec{v}$

Or $\vec{v} \times k\ \vec{u}$ if $\vec{u} \times k\ \vec{v}\ then\ \vec{u}\ and\ \vec{v}\ are\ same\ direction$

$if\ k > 0\ also\ \vec{u}\ and\ \vec{v}\ are\ in\ opposite\ direction\ if$

$k < o$

10. Addition of two vectors:

Addition of two vectors is explained by following two laws.

11. Triangle law of addition:

If two vectors $\underline{a}$ and $\underline{b}$ $are\ represented\ by\ two$

Sides $AB\ and\ BC\ \ of\ \Delta ABC$ s. that the terminal point $\underline{a}$ $coinside\ with\ the\ initial\ point\ of\ \underline{b}$.

Then $the\ third\ side\ AC\ of\ the\ triangle\ gives$

Vector sum AC of the triangle gives vector sum $\underline{a} + \underline{b}$

$$i.e\ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\ \Rightarrow \underline{a} + \underline{b} = \overrightarrow{AC}$$

12. Position vector:

The vector whose initial point is at called position vector of point p.

Position vector of $p = \overrightarrow{OP}$

$$\overrightarrow{AB} = \overline{\boldsymbol{DC}} = \vec{\boldsymbol{a}}$$

$$\overrightarrow{AB} = \overline{\boldsymbol{DC}} = \vec{\boldsymbol{a}}$$

13. Vectors in plane:

$let\ \overrightarrow{OP} = \underline{a}\ be\ a\ vector\ in\ xy = plane.\ we\ \ resolve\ x$

$and\ y\ be\ the\ horizonatal\ and\ vertical\ components$

$of\ \ a\ respectively. Then\ \underline{a} = [x, y]\ \ is\ a\ vector\ in\ xy - plane$

14. Parallelogram law of addition:

If two vectors $\underline{u}$ and $\underline{v}$ $are\ represented\ by\ two$

Sides $AB\ and\ AC\ of\ ||gram\ as\ shown\ in\ figure. then$

Diagonal $AD\ give\ the\ sum\ of\ \overrightarrow{AB}\ and\ \overrightarrow{AC}\ i.e$

$$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{AC} = \underline{u} + \underline{v}$$